Math 53 Discussion Problems Nov 5

1. The following integrals are expressed in terms of cylindrical coordinates. For each one of them, describe the region of integration and evaluate the integral.

   (a) $\int_0^{2\pi}\int_0^{3}\int_0^{z/3} r^3 dr dz d\theta$

   (b) $\int_0^{2}\int_{r-2}^{\sqrt{4-r^2}}\int_0^{2\pi} (r\sin\theta + 1) r d\theta dz dr$

2. The following integrals are expressed in terms of spherical coordinates. For each one of them, describe the region of integration and evaluate the integral.

   (a) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2} 3\rho^2 \sin\phi d\rho d\phi d\theta$

   (b) $\int_{\pi/6}^{\pi/3}\int_{\csc\phi}^{2\csc\phi}\int_0^{2\pi} \rho^2 \sin\phi d\theta d\rho d\phi$

3. Evaluate the integral $\iint_R (2x^2 - xy - y^2) dx dy$ where R is the region in the first quadrant bounded by the lines $y = -2x + 4, y = -2x + 7, y = x - 2, y = x + 1$, using the transformation $u = x - y, v = 2x + y$.

4. Evaluate the integral $\iint_R (\sqrt{\frac{y}{x}} + \sqrt{xy}) dx dy$ where R is the region in the first quadrant bounded by the hyperbolas $xy = 1, xy = 9$ and the lines $y = x, y = 4x$, using the transformation $x = \frac{u}{v}, y = uv$ with $u > 0, v > 0$.

5. Evaluate the integral $\iiint_D (x^2 y + 3xyz) dx dy dz$ where D is the region defined by the inequalities $1 \le x \le 2, 0 \le xy \le 2, 0 \le z \le 1$, using the transformation $u = x, v = xy, w = 3z$.